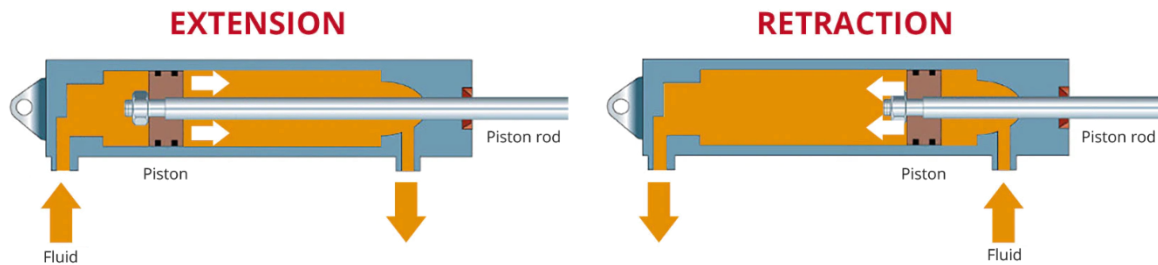


Picometer Hydraulic Pistons

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Conceptually, molecular hydraulic pistons might be able to achieve 20 pm accuracy. The basic design of a hydraulic piston is illustrated here:



Assume we wish to extend the piston rod a distance d to the right. Also assume the piston has radius R and the piston rod has radius r . Then we must inject a volume of working fluid $\pi R^2 d$ to the left of the piston into the left port and remove a volume of working fluid $\pi (R^2 - r^2) d$ from the right of the piston through the right port.

To retract the piston rod a distance d to the left, we reverse this process. We remove a volume of working fluid $\pi R^2 d$ from the left of the piston through the left port and inject a volume of working fluid $\pi (R^2 - r^2) d$ to the right of the piston through the right port.

If $d = .02 \text{ nm}$ (20 pm), $R = 2 \text{ nm}$ (the piston has a diameter of 4 nm), and $r = 1 \text{ nm}$ (the piston rod has a diameter of 2 nm), then to extend the piston rod 20 pm we inject $\sim 0.25 \text{ nm}^3$ of working fluid into the left port, and extract $\sim 0.19 \text{ nm}^3$ of working fluid from the right port.

If the working fluid is neon at 100K and a working pressure of 10^8 Pa (1 atm $\sim 100,000 \text{ Pa}$), the phase diagram for neon shows it is a supercritical fluid. [At \$10^8 \text{ Pa}\$, the density of Neon is \$50,823 \text{ mol/m}^3\$](#) , or $3.06 \times 10^{28} \text{ atoms/m}^3$, or ~ 30.6 Neon atoms per nm^3 .

Which means we add 8 atoms of neon through the left port and remove 6 atoms of neon through the right port to move the piston 20 pm to the right.

Of course, 8 and 6 are just integer approximations to the exact answer. If we design atomically precise pumps that actually pump exactly 8 and 6 neon atoms at a time, then to get things right we'd have to fine-tune the exact dimensions of the piston and the piston rod so that 8 and 6 were actually the exactly correct answers to the volume that needed to be pumped. Otherwise, as the piston moved its full throw the pressure and piston position might drift farther and farther from their correct values.

Now we need the compressibility of Neon at 100MPa (or its reciprocal, the bulk modulus). We can compute this from the [pressure/density curve](#). [At \$10^8 \text{ Pa}\$, the density of Neon is \$50,823 \text{ mol/m}^3\$](#) , which has a reciprocal of $1.9676 \times 10^{-5} \text{ m}^3/\text{mol}$. At $10^8 \text{ Pa} + 10 \text{ MPa}$, the density of Neon is $52,396 \text{ mol/m}^3$, with a reciprocal of $1.9085 \times 10^{-5} \text{ m}^3/\text{mol}$. This means 10MPa compressed the Neon at 100

MPa by 3%, giving it a bulk modulus at 100 MPa of $10^7/.03 = 0.33$ GPa. By comparison, the bulk modulus of water is ~ 2 GPa. Neon under these conditions is more compressible than water.

If we move the piston to the left by 20 pm, then the (maximum) volume of the 10 nm long piston is reduced by $.02 \text{ nm} / 10 \text{ nm} = .002 = 0.2\%$. In other words, we have compressed the working fluid, the neon, by 0.2%. As the bulk modulus is 0.33 GPa, this means the pressure required for this 0.002 compression must be 0.002 times 0.33 GPa = 0.66 MPa. As this pressure has to be generated by the piston, which has an area of $\pi R^2 = 4 \pi \sim 12.6 \text{ nm}^2$ or $12.6 \times 10^{-18} \text{ m}^2$, it takes a force of $0.66 \times 10^6 \text{ N/m}^2 \times 12.6 \times 10^{-18} \text{ m}^2 = 8.3 \text{ pN}$ to compress it this far. Work equals force times distance, so 8.3 pN times 20 pm gives an energy of $1.7 \times 10^{-22} \text{ J}$. This is well below thermal noise, so thermal vibration of this particular hydraulic system design will be substantially larger than our desired positional accuracy of 20 pm.

A piston using Neon as a working fluid, with a piston diameter of 4 nm, a piston rod with a diameter of 2 nm, a maximum throw of 10 nm, and operating at a pressure of 100 MPa, will not be able to maintain our desired positional accuracy of 20 pm in the face of thermal noise at 100K.

The bulk modulus of neon at 770 MPa can be computed from the pressure and density of neon at 770 MPa and 780 MPa, which is [available from NIST](#). The density of neon at 770 MPa is 85,335 mol/m³, at 780 MPa it is 85,577 mol/m³. Taking reciprocals, this is 0.00001172 m³/mol and 0.00001168 m³/mol. That is, an increase of 10 MPa from 770 MPa to 780 MPa caused the neon gas to shrink by 0.283%. This is a bulk modulus of $(10 \times 10^6)/0.00283 \sim 3.5$ GPa (larger than water).

Assuming our piston is 10 nm long (as before) and that the radius of the piston rod, r , is 1 nm (as before), but increasing the piston size to a radius of 10 nm (larger than the previously assumed value of 2 nm), and increasing our operating pressure from 100 MPa to 770 MPa (thereby increasing our bulk modulus to 3.5 GPa) we can repeat our calculations but get a different result. If the piston still moves 0.02 nm and the piston cylinder is still 10 nm long, we will still have compressed the working fluid by 0.2%, as before. The bulk modulus, however, is now 3.5 GPa, so the pressure (that is, the additional pressure above the 770 MPa working pressure) required for this compression is $3.5 \text{ GPa} \times 0.002 = 7 \text{ MPa}$. The surface area of the piston is $\pi R^2 = 314 \text{ nm}^2$. Computing the force on the piston that would be required for such a compression gives $7 \times 10^6 \text{ N/m}^2 \times 314 \times 10^{-18} \text{ m}^2 = 2,198 \times 10^{-12} \text{ N} = 2.2 \text{ nN}$. Work equals force times distance, but the average force is half the peak force, so $\frac{1}{2} \times 2.2 \text{ nN} \times 20 \text{ pm} = 22 \text{ zJ}$. (1 zJ is 10^{-21} J). Boltzmann's constant is $1.38 \times 10^{-23} \text{ J/K}$, so thermal noise at 100 K is 1.38 zJ. The energy required to compress the working fluid enough to create a 20 pm displacement of the piston would be 16 times kT at the 100 K operating temperature. This is an extremely unlikely event.

This hydraulic piston, operating at 100K, with a piston diameter of 20 nm, a working pressure of 770 MPa, and a maximum throw of 10 nm, should be more than able to provide a positional accuracy, in the face of thermal noise, that is significantly less than our 20 pm requirement.

Conclusion

A molecular hydraulic piston of ~ 20 nm diameter and using neon compressed to 770 MPa can achieve a positional accuracy of 20 pm with extremely high probability at 100 K.